RESEARCH ARTICLE   OPEN ACCESS

Edge Detection Using Directional Filtering

H. Aboelsoud M.
(Military Technical Colleague – Egyptian Armed Forces)

ABSTRACT
Edge detection can be performed by applying an edge filter in n directions. Conceptually, in order to estimate gradient, that is, to determine the edge map, eight filtering directions, e.g. (0, $\pi/8$, $\pi/4$, $3\pi/8$, $\pi/2$, $5\pi/8$, $3\pi/4$, $7\pi/8$), constitute the sufficient basis for the gradient calculation. Computationally, such a two-dimensional (2-D) eight-directional filter can be represented by a pair of real masks, that is, by one complex-number matrix. Final edges are determined after applying the non-maximum suppression followed by thresholding with hysteresis algorithms to obtain smooth and connected edge map.

Keywords **-** Conjugate images, directional filtering, edge detection.

I.   INTRODUCTION

An edge is characterized by an abrupt change in intensity indicating the boundary between two regions in an image. It is a local property of an individual pixel and is calculated from the image function in a neighborhood of the pixel. Edge detection is a fundamental operation in computer vision and image processing. It concerns the detection of significant variations of a grey level image. The output of this operation is mainly used in higher-level visual processing like three-dimensional (3-D) reconstruction, stereo motion analysis, recognition, scene segmentation, image compression, etc. Hence, it is important for a detector to be efficient and reliable [1].

In computer vision, edge detection is traditionally implemented by convolving the image with some form of linear filter, usually a filter that approximates a first or second derivative operator. An odd symmetric filter will approximate a first derivative, and peaks in the convolution output will correspond to edges (luminance discontinuities) in the image. An even symmetric filter will approximate a second derivative operator. Zero-crossings in the output of convolution with an even symmetric filter will correspond to edges [2].

If we ignore the noise present in images, edge detection can be based primarily on the computation of the gradient of intensity with subsequent thresholding of its magnitude. For this purpose the popular filters, like the Sobel filter [3], are used. Hence, a typical simple method of obtaining an edge

strength map is based on the estimation of two components of the intensity gradient vector using horizontal and vertical Sobel masks. Taking noise and edge imperfection into consideration, edge filters are traditionally constructed so as to improve the suppression of unwanted disturbances by appropriate lowpass filtering. The idea has originated perhaps from the concept of stochastic gradient [4] and from work of Marr and Hildreth [5], and in its current form was introduced by Canny [6] and developed further in many works using various criteria of optimality of the edge detection process. Following a classification of the optimal one-dimensional (1-D) edge filters presented by Heijden [7], the following operators are considered to be the main contenders: Canny [6] and its version by Deriche [8], Shen–Castan [9], [10], (see also [11]), Sarkar–Boyer [12], Boie–Cox [13], Spacek [14], Petrou-Kittler [15], and the CVM detector of Heijden [7]. The above filters were primarily constructed as 1-D filters and then extended appropriately into two dimensions.

This paper is focused on the use of directional filtering for edge detection. We show that, eventually, a 2-D eight-directional edge filter can be represented by a pair of matrix filters, or equivalently by one complex-number filter. We start with the description of the details of the eight-directional 2-D edge filter. In the next section we present experimental results of its application for edge detection in real life images. Finally, a conclusion is presented in Section-4.

II.   Eight-Directional Filter

Consider a digital grey level image F(x,y), assume we wish to convolve the image with a 2D Gaussian filter to smooth it. The 2D Gaussian smoothing operator G(x,y) is given by:

$$G(x,y) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x^2+y^2)}{2\sigma^2}} \qquad (1)$$

where x,y are the image co-ordinates and σ is a standard deviation of the associated probability distribution [16].

Let f(x,y) denotes to the smoothed image, then:

$$f(x,y) = F(x,y) \otimes G(x,y) \qquad (2)$$

where $\otimes$ denotes to the convolution operation. Consider a neighborhood R of a pixel (x,y) expanded in the direction d as shown in Fig. 1.   A general nonlinear filtering operation over the region R may now be defined by [17]:

$$f(x,y) \rightarrow g((x,y);R) = \int_R h(f(x+u_1, y+u_2),u)\partial u \qquad (3)$$

where f(x,y) and g((x,y);R) are the smoothed and filtered images respectively, such a region of interest is rotated in the directions(d₁; ……. ; dₙ) to cover the whole 2π angle.
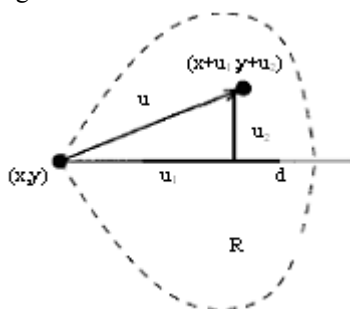

Fig. 1 Directional neighborhood of a pixel (x,y)

Applying the filtering operation (3) over the region R_{θ_k}; we obtain a so-called conjugate image, g_k (x,y). The edge strength may now be calculated by performing a vector addition of conjugate images. Using the complex-number notation, we have:

$$g_m(x, y) = \sum_{k=1}^{n} g_k(x, y)e^{j\theta_k} \qquad (4)$$

where g_m(x,y) is a complex-number-valued edge strength and its magnitude specifies a scalar edge strength.

If we use linear filters, then a conjugate image in the kth direction is calculated as a convolution of the smoothed image, f(x,y); with a kth directional filter, h_{θ_k}, as follows:

$$g_k(x,y) = f(x,y) \otimes h_{\theta_k}(x,y) \qquad (5)$$

Now, the complex edge strength can be obtained as a convolution of the smoothed image, f(x,y); with the complex edge filter h(x,y), as follows:

$$g_m(x,y) = \sum_{k=1}^{n} (f(x,y) \otimes h_{\theta_k}(x,y))e^{j\theta_k} \qquad (6)$$

$$g_m(x,y) = f(x,y) \otimes \sum_{k=1}^{n} h_{\theta_k}(x,y)e^{j\theta_k} \qquad (7)$$

$$g_m(x,y) = f(x,y) \otimes h(x,y) \qquad (8)$$

where h(x,y) is a sum of appropriately rotated filter components.

In order to estimate two components of the intensity gradient, eight filtering directions (aiming at detecting edges in all directions), e.g. (0, π/8, π/4, 3π/8, π/2, 5π/8, 3π/4, 7π/8), might be required. In this case, there are eight directions, θ_k=k π/8, k=0,1,2,….,7, thus eight filtering regions.

The complex "eight-directional" edge filter, h; can be directly determined using (7) by adding eight uni-directional components, namely:

$$h = h_0 + h_{\pi/8}e^{j\pi/8} + h_{\pi/4}e^{j\pi/4} + h_{3\pi/8}e^{j3\pi/8}$$
$$+ h_{\pi/2}e^{j\pi/2} + h_{5\pi/8}e^{j5\pi/8} + h_{3\pi/4}e^{j3\pi/4} + h_{7\pi/8}e^{j7\pi/8} \qquad (9)$$

for the simplicity let "a" denotes to $e^{j\pi/8}$, then:

$$h = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -a^{-1} & 0 & a^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.a + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.a + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.a$$

$$+ \begin{bmatrix} 0 & 0 & 0 & a^2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -a^2 & 0 & 0 & 0 \end{bmatrix}.a + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -a^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.a + \begin{bmatrix} 0 & j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -j & 0 \end{bmatrix}.a$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & ja & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -ja & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.a + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ ja^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -ja^2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.a \qquad (10)$$

Then the complex edge filter can be calculated as:

$$h = \begin{bmatrix} 0 & j & 0 & a^2 & 0 \\ ja^2 & ja & a^3 & a & 1 \\ 0 & -a^{-1} & 0 & a^{-1} & 0 \\ -1 & -a & -a^3 & -ja & -ja^2 \\ 0 & -a^2 & 0 & -j & 0 \end{bmatrix}.a \qquad (11)$$

$$h = \begin{bmatrix} 0 & 0 & 0 & 0.7071 & 0 \\ -0.7071 & -0.3827 & 0.3827 & 0.9239 & 1 \\ 0 & -0.9239 & 0 & 0.9239 & 0 \\ -1 & -0.9239 & -0.3827 & 0.3827 & 0.7071 \\ 0 & -0.7071 & 0 & 0 & 0 \end{bmatrix} +$$

$$j\begin{bmatrix} 0 & 1 & 0 & 0.7071 & 0 \\ 0.7071 & 0.9239 & 0.9239 & 0.3827 & 1 \\ 0 & 0.3827 & 0 & -0.3827 & 0 \\ 0 & -0.3827 & -0.9239 & -0.9239 & -0.7071 \\ 0 & -0.7071 & 0 & -1 & 0 \end{bmatrix} \quad (12)$$

From equations (11), and (12), a 2D eight-directional edge filter can be represented by a pair of matrix filters, or, equivalently by one complex-number filter. Fig. 2 shows its frequency response.
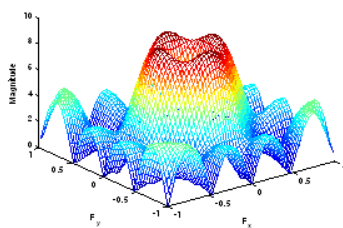

Fig. 2 Frequency response of eight-directional filter

Equation (12) can be written as:

$$h(x,y) = h_x(x,y) + j\, h_y(x,y) \quad (13)$$

From equation (8), the edge strength is given by:

$$g_m(x,y) = f(x,y) \otimes (h_x(x,y) + j\, h_y(x,y)) \quad (14)$$

$$g_m(x,y) = (f(x,y) \otimes (h_x(x,y))$$
$$+ j\, (f(x,y) \otimes h_y(x,y)) \quad (15)$$

$$g_m(x,y) = f_x(x,y) + j\, f_y(x,y) \quad (16)$$

its magnitude is given by:

$$mag(g_m(x,y)) = \sqrt{(f_x(x,y))^2 + (f_y(x,y))^2} \quad (17)$$

and its direction by $\tan^{-1}\left(\dfrac{f_y(x,y)}{f_x(x,y)}\right)$.

III. EXPERIMENTAL RESULTS

The eight-directional edge filter has been tested with three images: Wall image (Fig. 3), X-ray image of brain (Fig. 4), and CT image of abdomen (Fig. 5), and some results are presented in Fig. 6, Fig.7, and Fig.8 respectively.
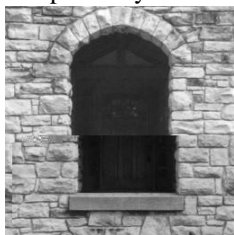

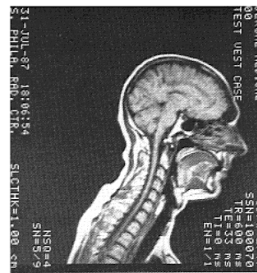Fig. 3 A 256 ×256 Wall image.

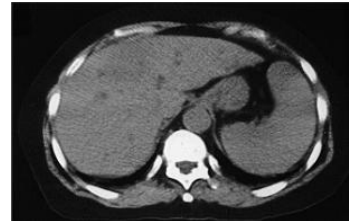Fig. 4 X-ray image of brain

Fig. 5 CT abdomen image

Fig. 6. (a): magnitude of the edge strength obtained using eight-directional filter of (12). (b): edge map after non-maximum and thresholding operations (lower thresh = 0.08, and upper thresh =0.2).
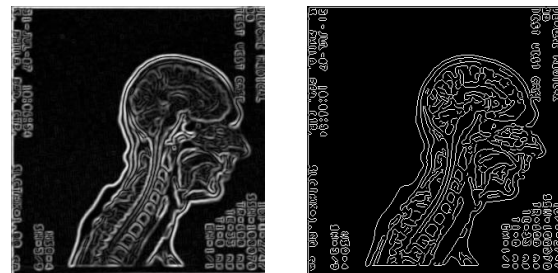

Fig. 7. (a): magnitude of the edge strength obtained using eight-directional filter of (12). (b): edge map after non-maximum and thresholding operations (lower thresh = 0.06, and upper thresh =0.15).
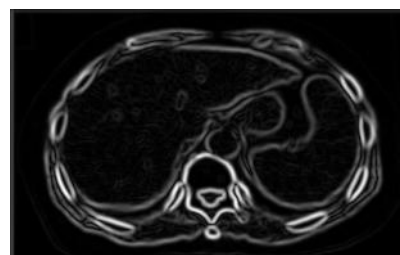

(a)

(b)

Fig. 8 (a): magnitude of the edge strength obtained using eight-directional filter of (12). (b): edge map after non-maximum and thresholding operations (lower thresh = 0.048, and upper thresh =0.12).

Figures 6(a), 7(a), and 8(a), illustrate the edge strength maps generated using convolution of images with the eight-directional filter of (12). In Figures 6(b), 7(b), and 8(b), the above maps are presented after operations of the non-maximum suppression followed by thresholding with hysteresis algorithms. The threshold parameters are manually adjusted for best results. It is clear from the figures, that the edge maps generated using convolution of images with the eight-directional filter of (12) contain many visibly important edges.

IV. CONCLUSION

Physical edges are one of the most important properties of objects. They correspond to object boundaries or to changes in surface orientation or material properties. Edges help to extracting useful information and characteristics of an image.

In this paper, we have discussed how to build a 2-D edge filter that collects information from eight directions around a central pixel. We show that, such a 2-D eight-directional filter can be represented by a pair of real masks, that is, by one complex-number matrix.

The above filter has been tested in the preceding section. The test is performed on three real life images. The final edges are determined after operations of the non-maximum suppression followed by thresholding with hysteresis algorithms. It is to be noted that on all the real life images considered, the eight-directional edge filter produced fairly good results.

REFERENCES

[1] Rishi R. Rakesh, Probal Chaudhuri, and C. A. Murthy 'Thresholding in Edge Detection: A Statistical Approach' *IEEE Transactions on image processing, vol. 13, no. 7, July 2004.*

[2] E. Davies, "Machine Vision: Theory, Algorithms and Practicalities", *Academic Press, pp 101 – 110, 1990.*

[3] R. M. Haralick and L. S. Shapiro, *Computer Vision, vol. 1. Reading, MA: Addison-Wesley, 1992.*

[4] A. K. Jain, Fundamentals of Digital Image Processing. *Englewood Cliffs, NJ: Prentice-Hall, 1989.*

[5] D. Marr and E. Hildreth, "Theory of edge detection," *in Proc. R. Soc. Lond. B, vol. 207, pp. 187–217, 1980.*

[6] J. Canny, "A computational approach to edge detection," *IEEE Trans .Pattern Anal. Machine Intell., vol. PAMI-8, pp. 679–698, Nov. 1986.*

[7] F. van der Heijden, "Edge and line feature extraction based on covariance models," *IEEE Trans. Pattern Anal. Machine Intell., vol. 17, pp. 16–33, Jan. 1995.*

[8] R. Deriche, "Using Canny's criteria to derive a recursively implemented optimal edge detector," *Int. J. Comput. Vis., vol. 1, pp. 167–187, 1987.*

[9] J. Shen and S. Castan, "An optimal linear operator for edge detection," *CVGIP: Graph. Models Image Processing, vol. 54, pp. 112–133, Mar. 1992.*

[10] "Toward the unification of band-limited derivative operators for edge detection," *Signal Process., vol. 31, pp. 103–119, 1993.*

[11] K. R. Rao and J. Ben-Arie, "Optimal edge detection using expansion matching and restoration," *IEEE Trans. Pattern Anal. Machine Intell., vol. 16, pp. 1169–1182, Dec. 1994.*

[12] S. Sarkar and K. L. Boyer, "On optimal infinite impulse response edge detection filters," *IEEE Trans. Pattern Anal. Machine Intell., vol. 13, pp. 1154–1171, Nov. 1991.*

[13] R. Boie and I. Cox, "Two dimensional optimum edge recognition using matched and Wiener filters for machine vision," *in Proc. Int. Conf. Computer Vision, London, U.K., 1997, pp. 450–456.*

[14] L. A. Spacek, "Edge detection and motion detection," *Image Vis. Comput., vol. 4, pp. 43–56, Feb. 1986.*

[15] M. Petrou and J. Kittler, "Optimal edge detectors for ramp edges," *IEEE Trans. Pattern Anal. Machine Intell., vol. 13, pp. 483–491, May 1991.*

[16] Milan Sonka, Vaclav Hlavac, and Roger Boyle, "Image Processing, Analysis and Machine Vision," 1993.

[17] Andrew P. Paplinski, "Directional Filtering in Edge Detection," *IEEE Tran. Image Proc., Vol. 7, No. 4, pp. 611-615, APRIL 1998.*